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Large Amplitude $m = 1$ Diocotron Mode Measurements in the Electron Diffusion Gauge Experiment¹

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Abstract. Smaller-diameter pure electron plasmas are generated in the Electron Diffusion Gauge (EDG) using a thoriated tungsten filament wound into a spiral shape with an outer diameter which is $1/4$ of the trap wall diameter. The $m = 1$ diocotron mode is excited in the plasma by means of the resistive-wall instability, using a resistor-relay circuit which allows the mode to be induced at various initial amplitudes. The dynamics of this mode may be predicted using linear theory when the amplitude is small. However, it has been observed [e.g., Fine *et al.*, Phys. Rev. Lett. **63**, 2232 (1989)] [1] that at larger amplitudes the frequency of this mode (relative to the small-amplitude frequency) exhibits a quadratic dependence on the mode amplitude. In this paper, the frequency shift and nonlinear dynamics of the $m = 1$ diocotron mode in the EDG device are investigated.

In this paper theoretical and experimental results relating to the nonlinear dynamics of the $m = 1$ diocotron mode in the standard Malmberg-Penning trap configuration are discussed. The experimental apparatus is first briefly described. The linear, infinite-length theory of this mode is then summarized for reference purposes, together with a qualitative summary of the effects of finite plasma length on the mode dynamics. The resistive-wall instability is then discussed as a means of exciting the desired modes in the experiment. It is shown that the observed growth rate of this instability is consistent with theoretical predictions and past experiments. Additionally, it is shown that the instability provides a means to excite the $m = 1$ mode to amplitudes large enough that a change in scaling of the mode frequency shift with amplitude is observed.

EXPERIMENTAL APPARATUS

The Electron Diffusion Gauge (EDG) was constructed to study the interaction of a pure electron plasma with background neutral gases [2]. It follows the standard Malmberg-Penning trap configuration, consisting of a cylindrical conducting shell subdivided axially into rings. The EDG has six rings of radius $R_w = 2.54$ cm; the rings vary from two to four inches in axial length, and one of them is azimuthally subdivided into two half-cylinders. A pure electron plasma is confined axially within the cylinder configuration

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by applying a large negative potential (typically ≈ -147 V) between two nonadjacent rings. Radial confinement is provided by a magnetic field ($B = 300 - 600$ G) aligned with the cylinder axis, and the plasma is created by thermionic emission of electrons from a thoriated tungsten filament. Typical plasmas in EDG have central number density $n_0 \approx 10^7$ /cm³, axial line density $N_L \approx 3 \times 10^7$ /cm, length $L_p \approx 15$ cm, temperature $T = 1 - 2$ eV, and column radius $R_p = 0.64 - 1.27$ cm.

LINEAR THEORY OF DIOCOTRON MODES

Using cylindrical coordinates, and assuming a system of infinite extent parallel to the magnetic field $B\hat{z}$, consider a single-species plasma within a conducting cylindrical shell of radius R_w . The linear theory of diocotron modes is developed by making the assumption that the motion perpendicular to the magnetic field corresponds to an $\mathbf{E} \times \mathbf{B}$ drift, where \mathbf{E} is the electric field created by the plasma. Treating the plasma as an incompressible fluid, and using the continuity and Poisson equations, gives the closed system of equations

$$\mathbf{u} = \frac{-\nabla\Phi \times \hat{z}}{B}, \quad \frac{\partial n}{\partial t} + \mathbf{u} \cdot \nabla n = 0, \quad \nabla^2\Phi = -\frac{qn}{\epsilon_0}, \quad (1)$$

for the three physical quantities \mathbf{u} , n , and Φ , which are, respectively, the fluid velocity, particle number density, and electric potential. In Eq. (1) ϵ_0 is the permittivity of free space, and q is the charge of the confined particles including the sign.

Assuming that the temporal and angular dependence of the linearized quantities varies as $\exp i(m\theta - \omega t)$, the above equations are linearized to give the eigenvalue equation [3, 4] for a plasma of infinite length

$$[\omega - m\omega_0(r)] \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \Phi_1}{\partial r} \right) - m^2 \frac{\Phi_1}{r^2} \right] = \frac{qm\Phi_1}{\epsilon_0 r B} \frac{\partial n_0}{\partial r}. \quad (2)$$

where $\omega_0(r) \equiv (rB)^{-1} \partial \Phi_0 / \partial r$. For $m = 1$ (assumed for the remainder of this paper), Eq. (2) admits simple solutions for the perturbed potential eigenfunction and eigenfrequency [5] of the form

$$\Phi_1 \propto r[\omega - \omega_0(r)] \quad , \quad \omega = \frac{|q|}{2\pi\epsilon_0} \frac{N_L}{R_w^2 B} \equiv \omega_\infty. \quad (3)$$

The mode frequency is obtained by using Gauss' Law in conjunction with the boundary condition imposed by the conducting cylinder, namely, that $\Phi_1(R_w) = 0$. The quantity N_L is the axial line density (the number density per unit axial length) of the plasma.

Physically, the $m = 1$ mode may be described as a combination of two rotational motions. The first is the equilibrium motion, i.e., the azimuthal $\mathbf{E} \times \mathbf{B}$ rotation of the plasma column around its own axis. The second is the perturbed motion, wherein the entire plasma column is displaced slightly from the z -axis, and precesses around the axis at frequency ω_∞ .

An early theoretical treatment [6] of the effects of finite plasma length predicts that the mode frequency ω should be shifted slightly upward, by a quantity which depends on the

ratio of the plasma radius R_p to the trap wall radius R_w . More recent work [7] modifies this hypothesis to include the dependence of the shift on the plasma temperature T and axial number density N_L . For typical EDG operating parameters, both theories predict a frequency shift of 10–20%, which is consistent with observations [8].

THE RESISTIVE-WALL INSTABILITY

It can be shown that the $m = 1$ diocotron mode is a negative-energy mode, which grows in amplitude as energy in the plasma is dissipated. This result can be obtained from a simple image charge calculation, modeling the plasma as a line charge with linear charge density $\lambda = qN_L$. The line charge is inside and parallel to the axis of a conducting cylinder of radius R_w , but displaced from the cylinder axis by a distance D . The equivalent image problem replaces the conducting cylinder by another line charge with charge density $-\lambda$, displaced from the original cylinder axis by a distance R_w^2/D along the ray from this axis to the original charge λ .

When $D \ll R_w$, consistent with linear theory, the electric field of the image charge causes a radial force per unit length on the plasma. Integration yields the energy per unit length when the plasma is displaced from the axis, i.e.,

$$\frac{F}{l} \approx \frac{q^2 N_L^2 D}{2\pi\epsilon_0 R_w^2} \quad \Rightarrow \quad \frac{W}{l} \approx -\frac{q^2 N_L^2 D^2}{4\pi\epsilon_0 R_w^2}. \quad (4)$$

The relevant physics in the expression for the energy W is contained in the negative sign and the dependence of the mode energy on D^2 . Displacing the plasma from the cylinder axis in the presence of dissipation excites the mode.

The plasma's motion induces image currents in the trap walls, which are ordinarily grounded (except for the confining end potentials). Consequently, dissipation (and thus an instability) may be introduced in the system by connecting a resistor R between a section of the trap wall and ground. Assuming that such a section has axial length L_s , and spans the azimuthal angle $\Delta\theta$, White *et al.* [9] have calculated the growth rate of the mode (which is proportional to the real part of the impedance between the trap section and ground) as

$$\gamma = \frac{4\epsilon_0 L_s^2}{\pi L_p} \omega^2 \sin^2\left(\frac{\Delta\theta}{2}\right) \left[\frac{R}{1 + \omega^2 R^2 C^2} \right]. \quad (5)$$

where C is the inherent capacitance of this section of the trap relative to the rest of the system, L_p is the plasma length, and ω is the mode frequency.

In EDG, a section of the trap wall of length $L_s = 5.08$ cm and azimuthal span $\Delta\theta = \pi$ is used to resistively grow the $m = 1$ diocotron mode, keeping the magnetic field and plasma line density constant in order to maintain a constant mode frequency $\omega/2\pi \approx 39.6$ kHz (left graph) and 38.0 kHz (right graph). In each case the plasma length is $L_p \approx 15$ cm and the wall capacitance $C \approx 200$ nF. The data, shown in Figure 1, agrees very well with the theoretical growth rate calculation except at high resistances. One explanation for this effect could be that a stray path to ground with a large, but finite

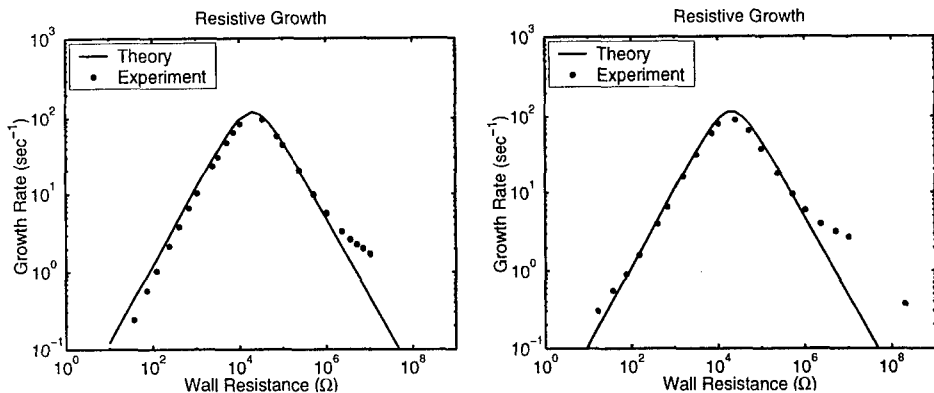


FIGURE 1. Measurements of the growth rate of the resistive-wall instability as a function of wall resistance, plotted together with the theoretical prediction of Eq. (5) (solid line). The data on the right was previously measured in EDG by Chao *et al.* [10].

resistance R_s exists between the wall sector and ground. Such a path would modify the impedance Z of this sector, such that

$$\text{Re}(Z) \propto \left[\frac{R}{1 + \omega^2 R^2 C^2} \right] \Rightarrow \text{Re}(Z) \propto \left[\frac{R(1 + R/R_s)}{(1 + R/R_s)^2 + \omega^2 R^2 C^2} \right]. \quad (6)$$

This results in a negligible change in the shape of the theoretical curve for $R_s \gg R$, but has a significant effect for $R_s \sim R$. The original growth rate, for $R \gg 1/\omega C$, decreases as $1/R$, but the asymptotic behavior of the new growth rate in the same regime is independent of R , i.e., it is simply a constant multiplying R_s . Choosing $R_s = 3.86 \text{ M}\Omega$ gives the best fit (in the least-squares sense) to the recent data, as shown in Figure 2. However, further investigation on this point is needed, particularly at very large resistances.

MODE GROWTH TO NONLINEAR AMPLITUDES

Using the resistive-wall instability as a means of exciting the $m = 1$ mode to large amplitudes, the nonlinear effects resulting from large amplitudes can be studied. The nonlinear problem of the dependence of the mode dynamics on mode amplitude was first treated theoretically by Prasad and Malmberg [11], who used a perturbation theory approach to show that for small displacements, the frequency of this mode should shift such that $(\omega - \omega_\infty)/\omega_\infty \propto (D/R_w)^2$, where D is the mode amplitude (displacement of the plasma column from the z -axis). This result was also obtained using a different model by Fine [12], and was experimentally investigated by Fine *et al.* [1], who give the empirical formula

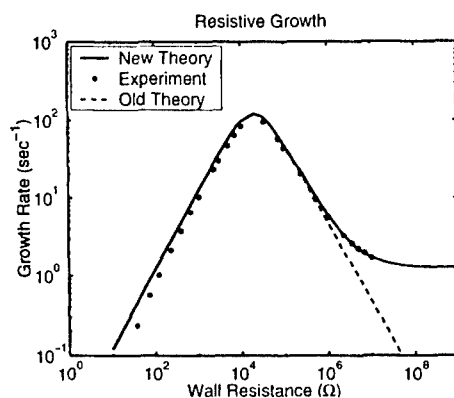


FIGURE 2. Measurements of the growth rate of the resistive-wall instability as a function of wall resistance, plotted together with the theoretical prediction of Eq. (5) (solid line) and the modified prediction of Eq. (6) (dashed line).

$$\frac{\omega - \omega_{\infty}}{\omega_{\infty}} = \left[1 - 7.3 \left(\frac{R_p}{R_w} \right)^6 \right] \left(\frac{D}{R_w} \right)^2 \quad (7)$$

for the scaling of the frequency shift as a function of mode amplitude.

Experiments aimed at verifying these results in EDG were performed by Chao [13], and the results of these experiments are shown in Figure 3. However, the radii of the plasmas used in these experiments were approximately equal to the filament radius $R_f = R_w/2 = 1.27$ cm, and as a result, the range of off-axis displacements in which

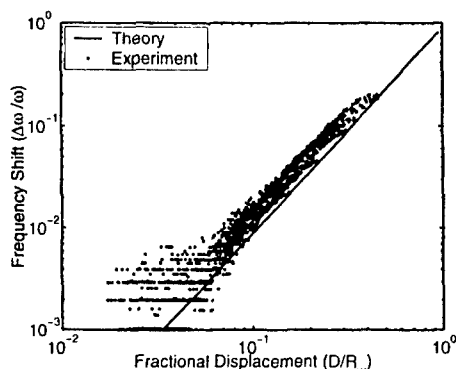


FIGURE 3. Data obtained by Chao [13] measuring the frequency shift $(\omega - \omega_{\infty})/\omega_{\infty}$ as a function of relative displacement (D/R_w) , together with the prediction of Fine *et al.* [1] (solid line). The scatter at low frequency shifts is due to nonideal measurement apparatus.

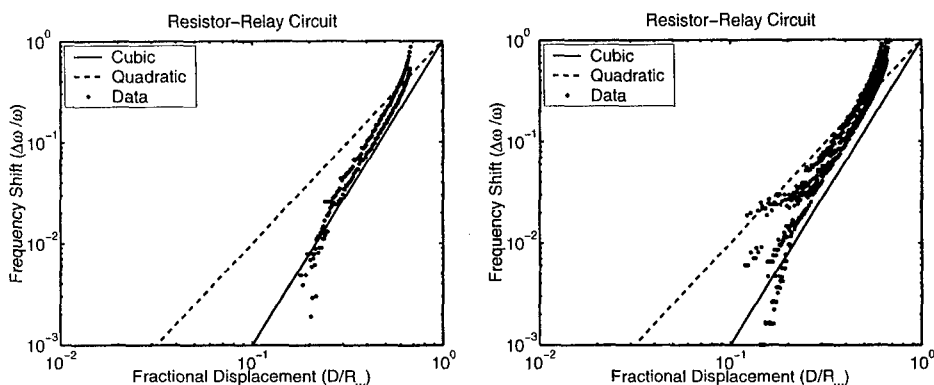


FIGURE 4. Measurements of the mode frequency shift as a function of relative mode displacement (D/R_w) for large relative displacements. Modes were excited using the relay circuit, with a resistance of 2.1 – 4.1 k Ω applied to the sectored trap wall for 0.2 seconds. Cubic (solid line) and quadratic (dashed line) dependences are plotted for reference. It is only the slope of these lines that is physically meaningful.

experiments could be performed was limited by eventual contact of the plasma with the trap wall. To test the scaling at large amplitudes, a smaller filament of radius $R_f = R_w/4 = 0.64$ cm was installed in EDG, and the resulting smaller-radius plasmas were excited by the resistive-wall instability to large relative amplitudes. Resistances were placed between the wall sector and ground for approximately 0.2 seconds, after which the wall sector was shorted to ground by a relay circuit. Modes of varying initial amplitude were obtained by changing the wall resistance, and measurements of the plasma displacement and frequency were subsequently taken as the mode damped.

In the present experiments, we find a persistent discrepancy between our data and the observation of Fine *et al.* [1] that the frequency shift scales quadratically in the relative mode amplitude. The data, shown in Figure 4, indicates that the dependence of this shift on amplitude is cubic for displacements on the order of the plasma diameter ($D/R_w \approx 0.3 - 0.5$). For larger displacements, the shift is proportional to successively higher powers of the displacement (limited by contact of the plasma with the trap wall). The behavior at high relative displacements may be consistent with the work of Fine *et al.*, who show that the frequency shift is an effect caused by distortion of the cylindrical shape of the plasma (due to interaction with the image charge distribution in the cylinder walls). Since the relative amplitudes in the experiments of Fine *et al.* were small ($D/R_w \leq 0.25$), this distortion was simply characterized by the quadrupole moment of the plasma. For larger relative displacements, however, the distortion is probably more complicated (thus requiring higher-order multipole moments for an accurate description). At low relative displacements, it is not clear why the scaling appears to be cubic rather than quadratic. Trap asymmetries may have caused the plasma to expand radially as the experiment progressed, which may have resulted in slight charge loss to the wall. The plasma's subsequent relaxation to a new quasi-equilibrium as the mode damped may have introduced physical effects which have not been considered previously.

CONCLUSION

In this paper, we have reviewed the linear theory of the $m = 1$ diocotron mode and the effects of finite plasma length on this theory. In addition, it has been shown that the $m = 1$ mode can be excited to large amplitudes by the resistive-wall instability, and that the growth rate of this instability agrees well with theoretical predictions. Finally, the nonlinear frequency shift of this mode at large amplitudes has been measured, and a cubic scaling of the frequency shift at large amplitudes (as opposed to the quadratic scaling observed at lower amplitudes) has been observed.

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